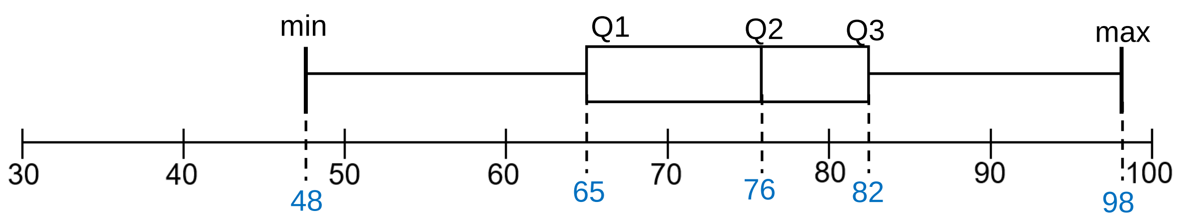
MATHEMATICS - PAPER 2 MEMORANDUM **GRADE 10 – NOVEMBER 2023**

QUESTION 1:

The Alex recycling club collected cans for a period of 23 school days. The number of cans collected each day was recorded and the data is shown below:

48	50	52	59	60	65	68	71
73	76	76	76	77	78	79	80
81	82	82	84	91	92	98	

1.1	Determine the median number of cans collected each day.	(1)
	Median $= 76$ ✓	
1.2	Determine the range of the data.	(1)
	Range $= 98 - 48$ $= 50$ ✓ answer	
1.3	Determine the Interquartile Range (IQR).	(2)
	$Q_1 = 65$ $Q_3 = 82$ ✓ for Q1 and Q3	
	$\text{IQR} = 82 - 65$	
	$= 17$ ✓ answer	
1.4	Draw a box-and-whisker diagram to represent the data.	(3)
	 ✓ min and max ✓ median ✓ Q1 and Q3	
1.5	The recycling club realises there was a mistake in the records. They actually collected 4 **less** cans per day. How will this impact the:	
	1.5.1 Range	(1)
	Remain the same ✓	
	1.5.2 Mean	(1)
	Will decrease by 4 ✓	
		[9]

QUESTION 2:

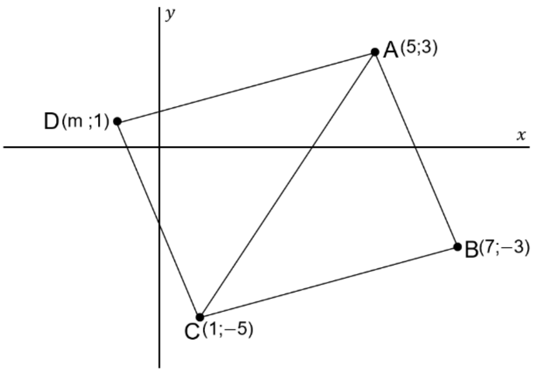
The heights, h, of the learners at Hogwarts High School in a Grade 10 class were measured and recorded as follows:

Height/length (in cm)	No. of learners $(f_{1)})$
$120 \leq x < 130$	5
$130 \leq x < 140$	6
$140 \leq x < 150$	11
$150 \leq x < 160$	13
$160 \leq x < 170$	5
Total	**40**

2.1	Write down the modal class for the data.	(1)
	$150 \leq x < 160$ ✓	
2.2	Determine the estimated mean for the data. Round off your answer to the **nearest cm.**	(3)
	$Estimated\ \bar{x} = \frac{5870}{40}$ ✓ numerator ✓ denominator	
	$= 146,75$ ✓ answer	
2.3	In which interval would the median of the data lie?	(1)
	$140 \leq x < 150$ ✓	
		[5]

QUESTION 3:

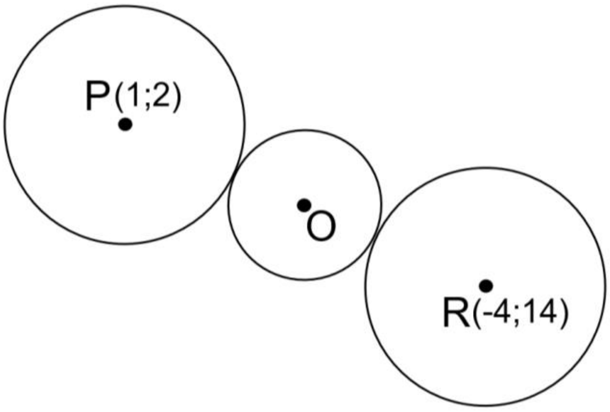
3.1	In the diagram below, $A(5\,;\,3), B(7\,;-3), C(1\,;-5)$ and $D(m\,;1)$ are the four vertices of a **quadrilateral** in the Cartesian plane.	



3.1.1	Determine the midpoint of AC.	(2)
	$M_{AC} = \left(\frac{5+1}{2};\frac{3+(-5)}{2}\right)$ ✓substitution	
	$M_{AC} = (3;-1)$ ✓ answer	
3.1.2	Determine the gradient of AB.	(2)
	$m_{AB} = \frac{3-(-3)}{5-7}$ ✓substitution	
	$m_{AB} = -3$ ✓answer	
3.1.3	Prove that AB⊥BC.	(3)
	$m_{BC} = \frac{-3-(-5)}{7-1}$ ✓substitution	
	$m_{BC} = \frac{1}{3}$ ✓	
	$m_{AB} \times m_{BC}$	
	$= -3 \times \frac{1}{3}$ ✓	
	$= -1$	
3.1.4	Hence, determine the area of ΔABC.	(4)
	$d_{AB} = \sqrt{(5-7)^2 + (3-(-3))^2}$	
	$= 2\sqrt{10}$ ✓	
	$d_{BC} = \sqrt{(1-7)^2 + (-5-(-3))^2}$	
	$= 2\sqrt{10}$ ✓	
	$Area\ of\ \Delta = \frac{1}{2}(2\sqrt{10})(2\sqrt{10})$ ✓	
	$= 20\ units^2$ ✓	

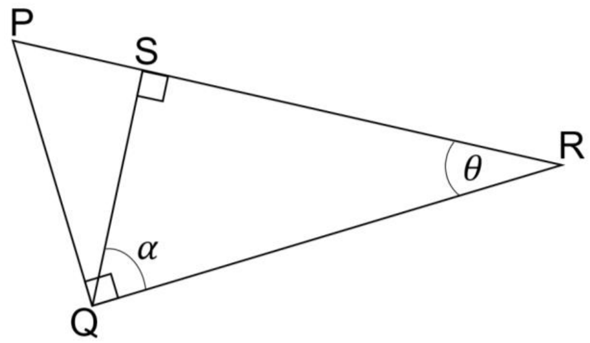
3.1.5	If AD‖BC, show that m = −1.	(2)
	$m_{AD} = m_{BC}$	
	$\frac{3-1}{5-m} = \frac{1}{3}$ ✓	
	$\therefore 2 = \frac{5}{3} - \frac{1}{3}m$	
	$\therefore \frac{1}{3} = -\frac{1}{3}m$ ✓	
	$\therefore m = -1$	

3.2	Two identical circles with centres P$(1\,;\,2)$ and R$(-4\,;\,14)$ touch a third circle with centre O as shown in the diagram below. P, O and R lie on the same straight line.	

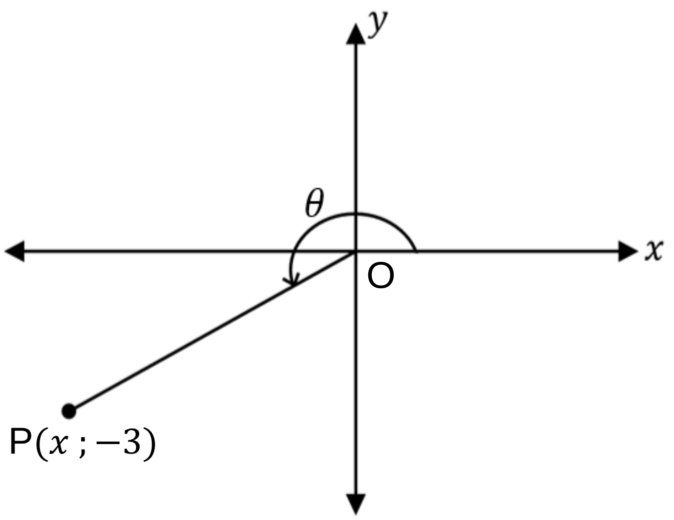


	Determine the length of the diameter of the third circle, with centre O, if the two identical circles each have a radius of 4,5 units each.	(3)
	$d_{PR} = \sqrt{(1-(-4))^2 + (2-14)^2}$ ✓	
	$= 13$ ✓	
	Diameter of smaller O $= 13 - 9 = 4$ ✓	
		[16]

QUESTION 4:

4.1	In the diagram below, ΔPQR is a right-angled triangle. PQ⊥QR and QS⊥SR. 	
4.1.1	Write down a ratio for $\tan\theta$ in the ΔPQR.	(1)
	$\tan\theta = \frac{PQ}{QR}$ ✓	
4.1.2	Write down the ratio for $\sec\alpha$.	(1)
	$\sec\alpha = \frac{QR}{QS}$ ✓	
4.2	Given: $\hat{A} = 112{,}4°$ and $\hat{B} = 48{,}6°$.	
4.2.1	Determine the value of $\sin(A - B)$.	(2)
	$\sin(112{,}4° - 48{,}6°)$ ✓ $= 0{,}90$ ✓	
4.2.2	Prove, using a calculator, that $\cos 2A = \cos^2 A - \sin^2 A$.	(3)
	LHS $= \cos 2A$ $= \cos 2(112{,}4°)$ ✓ substitution both $= -0{,}71$ ✓ RHS $= \cos^2 A - \sin^2 A$ $= \cos^2(112{,}4°) - \sin^2(112{,}4°)$ $= -0{,}71$ ✓	
4.3	WITHOUT USING A CALCULATOR, simplify as far as possible:	
	4.3.1 $\sqrt{3}\,\sin 60° - \cos 45° . \sin 45° - \sin 90°$	(5)
	$= \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$ ✓ $- \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$ ✓ $- (1)$ ✓ ✓✓✓ substitution of special ∠s $= \frac{3}{2} - \frac{1}{2} - 1$ ✓ simplification $= 0$ ✓ answer	

	4.2.2	$\dfrac{\cos^2(180° + x)\ .\ \tan(360° - x)}{\tan(180° - x)}$	(4)
		$= \dfrac{\cos^2 x ✓ . -\tan x ✓}{-\tan x ✓}$ ✓✓✓ reduction $= \cos^2 x$ ✓ answer	
4.3	In the diagram below, P is a point $(x\,;-3)$ in the third quadrant and OP = 5 units. θ is the angle from the positive x – axis to OP.		



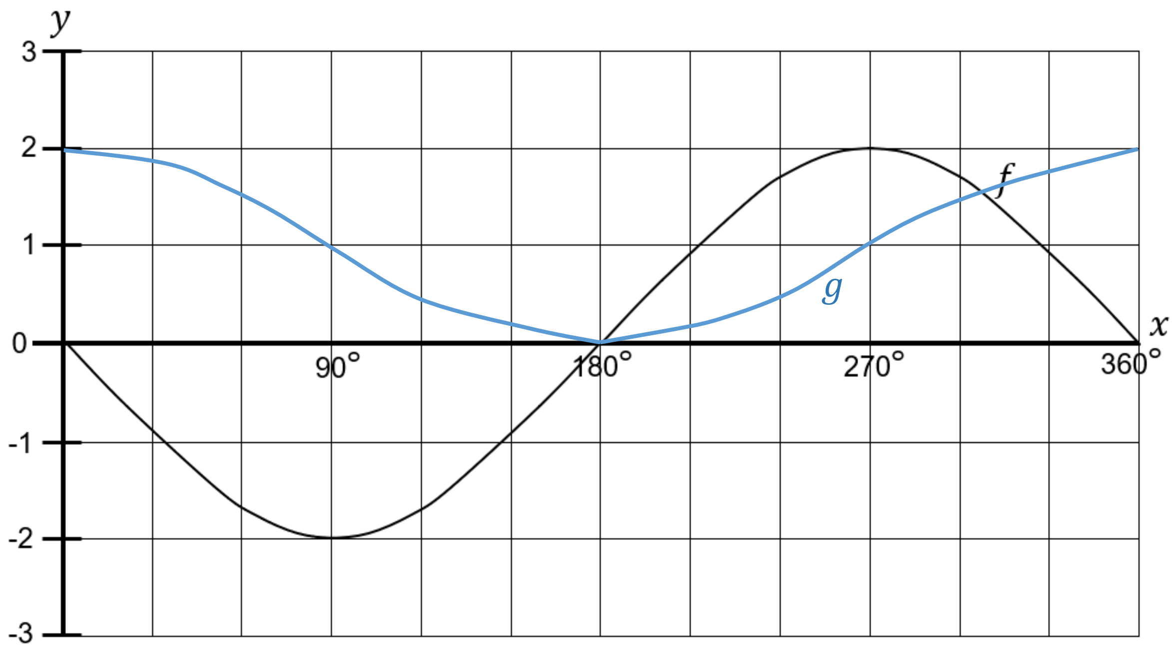
	4.3.1	Determine the value of x.	(2)
		$x = \pm\sqrt{(5)^2 - (-3)^2}$ (Pyth) ✓ $\therefore x = \pm 4$ $\therefore x = -4$ ✓	
	4.3.2	Calculate the value of $\cot\theta + \sin\theta$ WITHOUT the use of a calculator.	(3)
		$\cot\theta + \sin\theta$ $= \left(\frac{-4}{-3}\right) + \left(\frac{-3}{5}\right)$ ✓substitution $= \left(\frac{4}{3}\right) + \left(\frac{-3}{5}\right)$ $= \frac{20-9}{15}$ ✓ $= \frac{11}{15}$ ✓	
			[21]

QUESTION 5:

5.1	Solve for x, where $0° \leq x \leq 90°$. Give your answers correct to TWO decimal places.		
	5.1.1	$\operatorname{cosec} x + 1{,}4 = 3$	(3)
		$\operatorname{cosec} x = 1{,}6$ ✓ transposition	
		$\therefore \sin x = \frac{1}{1{,}6}$ ✓ reciprocal	
		$\therefore x = 38{,}68°$ ✓ answer	
	5.1.2	$3sin(x + 20°) = 2{,}952$	(3)
		$\therefore sin(x + 20°) = 0{,}984$ ✓ division by 3	
		$\therefore x + 20° = 79{,}7369 \ldots °$ ✓	
		$\therefore x = 59{,}74°$ ✓	
5.2	A Grade 10 Science class visited the weather tower as part of a class trip. During the trip Lindiwe (at D) and John (at B) were standing on opposite sides of the tower (AC) as shown in the diagram below. D, C and B lie on the same straight line. Lindiwe was standing 100m away from the base of the tower (C). The angle of depression from E to D is 68,2°.		
	DIAGRAM ON THE PAPER		
	5.2.1	Write down the size of $\hat{D}$. Give a reason for your answer.	(1)
		$\hat{D} = 68{,}2°$ $(als\ \angle s =; EA \parallel DC)$ ✓SR	
	5.2.2	Calculate the height of the tower, correct to the **nearest metre**.	(3)
		$\tan 68{,}2° = \frac{AC}{100}$ ✓	
		$\therefore AC = 100 \tan 68{,}2$ ✓	
		$\therefore AC = 250{,}01 \approx 250m$ ✓	
	5.2.3	If the distance DB between John and Lindiwe is 144m, determine the angle of elevation, θ, from John to the top of the tower. Round your answer off to the **nearest degree.**	(3)
		$BC = 144 - 100 = 44m$ ✓	
		$\therefore \tan \theta = \frac{250}{44}$ ✓	
		$\therefore \theta = 80{,}018 \ldots \approx 80°$ ✓	
			[13]

QUESTION 6:

6.1	The graph of $f(x) = -2\sin x$ is given. Sketch the graph of $g(x) = \cos x + 1$ on the set of axes given below, clearly indicating all turning points and intercepts with axes.	(3)



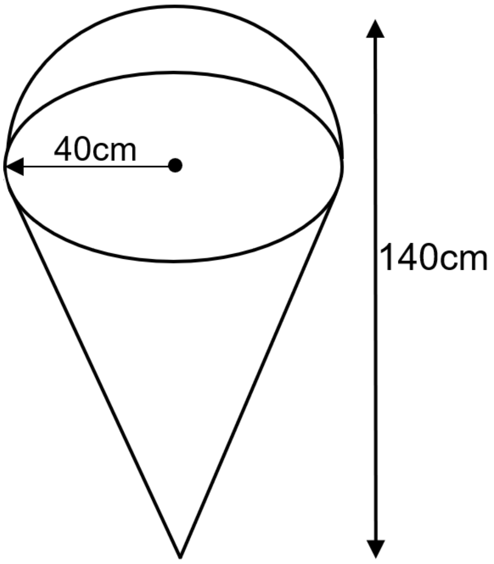
✓ end points ✓(90°;1) and (270°;1) ✓turning point (180°;0)

6.2	Write down the amplitude of $f(x)$.	(1)
	$Aplitude = 2$ ✓	
6.3	Write down the range of $g(x)$.	(2)
	$0 \leq y \leq 2$ or $y \in [0; 2]$ ✓✓Accuracy	
6.3	For which value(s) of x is $f(x)$ an increasing function ?	(2)
	$90° < x < 270°$ or $x \in (90°; 270°)$ ✓✓Accuracy	
6.4	For which value(s) of x will $f(x).g(x) \geq 0$	(2)
	$180° \leq x \leq 360°$ or $x \in [180°; 360°]$ ✓✓Accuracy	
6.5	Write down the equation of $h(x)$ if $h(x)$ is the reflection of $f(x)$ about the $x-$axis and then translated 1 unit upwards.	(1)
	$h(x)$: $y = 2\sin x + 1$ ✓Accuracy	
		[11]

QUESTION 7:

The model below shape is constructed by using a hemisphere and a cone.

The height of the model is 140 cm and the radius of the hemisphere is 40 cm.



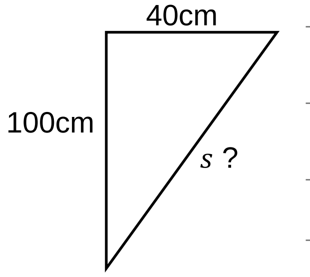
Available Formulae

$V = \frac{1}{3}\pi r^2 h$

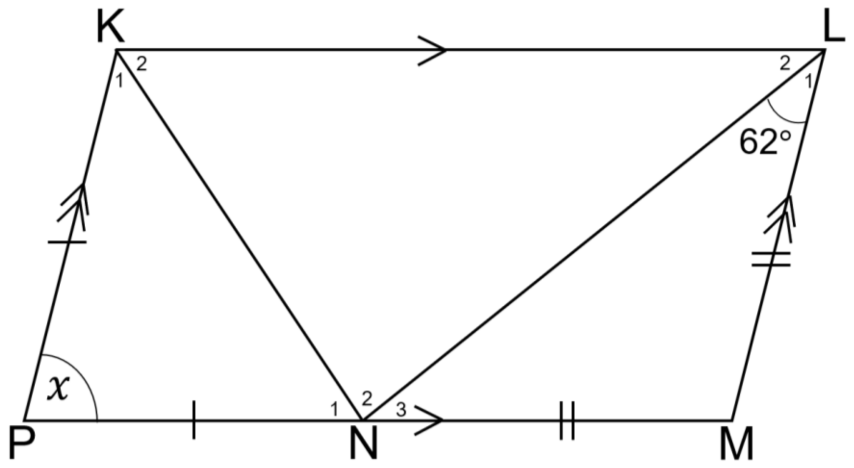
$V = \frac{4}{3}\pi r^3$

$SA = \pi r^2 + \pi rs$

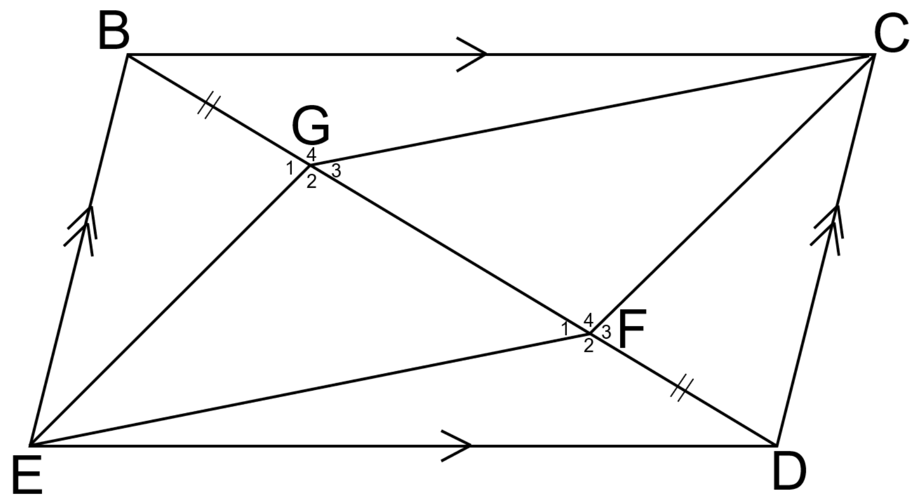
$SA = 4\pi r^2$

7.1	Calculate the volume of the model in cm^3.	(3)
	$V = \frac{1}{3}\pi r^2 h + \frac{1}{2}\left(\frac{3}{4}\pi r^3\right)$	
	$V = \frac{1}{3}\pi(40)^2(100)$ ✓ $+ \frac{1}{2}\left(\frac{3}{4}\pi(40)^3\right)$ ✓	
	$V = 301\ 592{,}89$ cm^3 ✓	
7.2	Calculate the total exterior surface area of the model in cm^2.	(4)
	$TSA = \pi rs + \frac{1}{2}(4\pi r^2)$	
	$TSA = \pi(40)(107{,}703\ ...)$ ✓ $+ \frac{1}{2}(4\pi(40)^2)$ ✓	
	$TSA = 23\ 587{,}49$ cm^2 ✓	
		
	$s = \sqrt{(100)^2 + (40)^2}$ $(Pyth)$	
	$s = 107{,}703\ ...$ ✓	
		[7]

QUESTION 8:

8.1	In the diagram below, **KLMP is a parallelogram**. LM = MN, KP = PN, $\hat{L}_1 = 62°$ and $\hat{P} = x$.	
		
8.1.1	Calculate the value of x.	(3)
	$\hat{N}_3 = 62°$ $(\angle s\ opp\ equal\ sides)$ ✓SR	
	$\hat{L}_2 = 62°$ $(als\ \angle s =; KL \parallel PM)$ ✓SR	
	$x = 124°$ $(opp\ \angle s\ of\ parm =)$ ✓SR	
8.1.2	Show through calculation that ΔKNL is a right-angled triangle.	(4)
	$\hat{N}_1 = \frac{180°-124°}{2}$ $(sum\ \angle s\ of\ \Delta)$ ✓ and $(\angle s\ opp\ equal\ sides)$ ✓	
	$\hat{N}_1 = 28°$ ✓	
	$\hat{N}_2 = 90°$ $(\angle s\ on\ a\ str\ line)$ ✓SR	

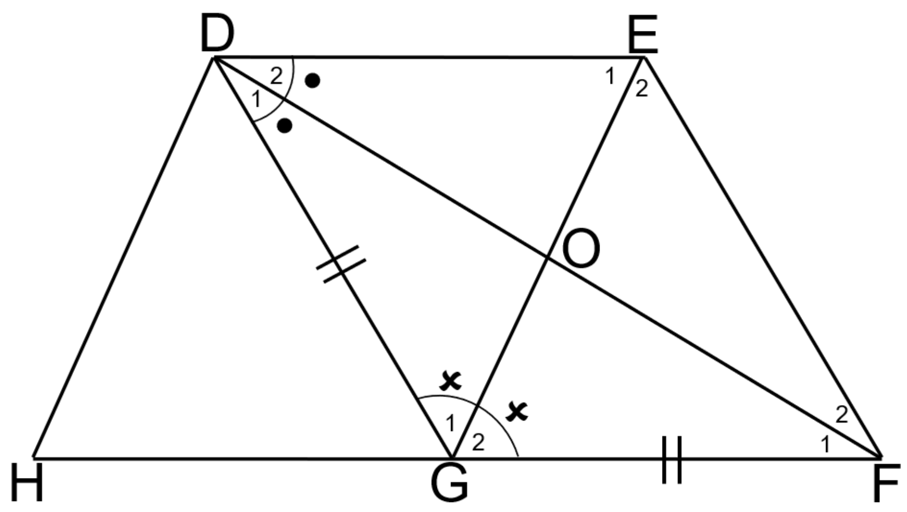
8.2	In the diagram below, BCDE is a parallelogram and BG = FD.	



8.2.1	Prove that ΔBGE ≡ ΔDFC.	(3)
	$In\ \Delta BGE\ and\ \Delta DFC$:	
	1. $BG = FD$ $(given)$	
	2. $E\hat{B}D = C\hat{D}B$ $(als\ \angle s =; EB \parallel CD)$ ✓SR	
	3. $EB = CD$ $(opp\ \angle s\ of\ parm =)$ ✓SR	
	$\therefore \Delta BGE \equiv \Delta DFC\ (SAS)$ ✓Reason	
8.2.2	Hence, or otherwise, prove that EG∥FC.	(3)
	$\hat{G}_1 = \hat{F}_3\ (proven\ thr \equiv)$ ✓	
	$\hat{G}_2 = \hat{F}_4\ (\angle s\ on\ a\ str\ line)$ ✓SR	
	$\therefore EG \parallel FC\ (alt\ \angle s =)$ ✓Reason	
		[13]

QUESTION 9:

In the diagram below, $\widehat{D}_1 = \widehat{D}_2$, $\widehat{G}_1 = \widehat{G}_2$ and $DG = GF$.



9.1	Prove that DEFG is a parallelogram.	(5)
	$\widehat{D}_1 = \widehat{F}_1$ $(\angle s\ opp\ equal\ sides)$ ✓SR	
	$\therefore DE \parallel FG$ $(alt\ \angle s =)$ ✓SR	
	$In\ \Delta DOG\ and\ \Delta DOE$:	
	1. $\widehat{D}_1 = \widehat{D}_2$ $(given)$	
	2. $\widehat{G}_2 = \widehat{G}_1 = D\widehat{E}G$ $(als\ \angle s =; DE \parallel FG)$ ✓SR	
	3. $DO = DO$ $(common)$	
	$\therefore \Delta DOG \equiv \Delta DOE\ (AAS)$ ✓SR	
	$\therefore DEFG\ is\ a\ parallelogram$ $(1\ pair\ opp\ sides = and\ parallel)$ ✓Reason	
		[5]