ALEXANDER ROAD HIGH SCHOOL

GRADE 10 MEMORANDUM

NOVEMBER 2023	**MATHEMATICS PAPER 1**	**2 HOURS**
EXAMINER: IC	**MODERATOR: LE**	**TOTAL: 100**

QUESTION 1

1.1 Convert the recurring decimal fraction $8,\dot{6}\dot{4}$ to a common fraction in its simplest form.

Let $8,\dot{6}\dot{4} = x$

$100x = 864,646464\ldots$ ✓
$-\ \ x = \ \ \ 8,646464\ldots$
$99x = 856$ ✓
$\therefore x = \frac{856}{99}$ ✓

(3)

1.2 Factorise the following expressions fully:

1.2.1 $21x^2 + 81xy + 54y^2$
$= 3(7x^2 + 27xy + 18y^2)$ ✓**CF**
$= 3(7x + 6y)(x + 3y)$ ✓✓

(3)

1.2.2 $x^3 + 27$
$= (x + 3)(x^2 - xy + 9)$ ✓✓

(2)

1.2.3 $18x^2 + 3x - 8y^2 + 2y$
$= 2(9x^2 - 4y^2) + 3x + 2y$✓
$= 2(3x + 2y)(3x - 2y)$✓ $+ (3x + 2y)$
$= (3x + 2y)[2(3x - 2y) + 1]$ ✓
$= (3x + 2y)(6x - 4y + 1)$ ✓

(4)

1.3 Simplify the following expressions fully:

1.3.1 $2x(x - 2)^2 - 2x(-3 + x^2)$
$= 2x(x^2 - 4x + 4)$✓ $+ 6x - 2x^3$✓
$= 2x^3 - 8x^2 + 8x + 6x - 2x^3$ ✓
$= -8x^2 + 14x$ ✓

(4)

1.3.2 $\dfrac{x^2+x-6}{3x^2-12x} \div \dfrac{x^3-2x^2}{x^2-16} \times \dfrac{3}{x+4}$

$= \dfrac{(x+3)(x-2)}{3x(x-4)} \times \dfrac{(x+4)(x-4)}{x^2(x-2)} \times \dfrac{3}{x+4}$ ✓ ✓ ✓ ✓

$= \dfrac{x+3}{x^3}$ ✓ (5)

1.3.3 $\dfrac{49^x-9}{7^x+3}$

$= \dfrac{7^{2x}-9}{7^x+3}$ ✓7^{2x}

$= \dfrac{(7^x+3)(7^x-3)}{7^x+3}$ ✓

$= 7^x - 3$ ✓ (3)

1.4 Solve for x:

1.4.1 $12x^2-4x-21=0$

$= (2x-3)(6x+7)$ ✓

$\therefore x = \frac{3}{2} or -\frac{7}{6}$ ✓✓ (3)

1.4.2 $\dfrac{x^2+3}{c^2} = b^2$

$x^2+3 = b^2c^2$ ✓

$x^2 = b^2c^2-3$ ✓

$\therefore x = \pm\sqrt{b^2c^2-3}$ ✓ (3)

1.4.3 $\dfrac{-5x}{3} \geq x+8$

$-5x \geq 3x+24$ ✓

$-8x \geq 24$ ✓

$\therefore x \leq -3$ ✓**A** (3)

1.4.4 Show that $\dfrac{5^{2024}-5^{2023}}{5^{2022}} = 20$

$LHS = \dfrac{5^{2022}(5^2-5^1)}{5^{2022}}$ ✓5^{2022} ✓(5^2-5^1)

$= 25-5$ ✓

$= 20 = RHS$ (3)

1.5 Solve for x and y simultaneously in the following equations:

$x = 4 - 4y$ and $3x + 2y = 2$

$3(4 - 4y) + 2y = 2$ ✓
$12 - 12y + 2y = 2$
$-10y = -10$ ✓
$\therefore y = 1$ ✓

$x = 4 - 4(1)$
$\therefore x = 0$ ✓ (4)

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QUESTION 2

2.1 Consider the finite linear number pattern 17; 14; 11; ……; -103.

2.1.1 Write down the fourth term (T_4) of the pattern.

$T_4 = 8$ ✓ (1)

2.1.2 Determine the general term (T_n) of the pattern.

$T_n = -3n$✓ $+ 20$ ✓ (2)

2.1.3 Calculate the number of terms in the pattern if -103 is the last term.

$-103 = -3n + 20$ ✓
$3n = 123$ ✓
$\therefore n = 41$ ✓ (3)

2.1.4 Which term in the pattern will be the first term to have a negative value? Show all your working.

$-3n + 20 < 0$ ✓
$-3n < -20$
$n < 6,\dot{6}$ ✓
$\therefore n = 7$ ✓ (3)

2.2 Given:

- The common difference of a linear number pattern is -5.
- The nineteenth term (T_{19}) is -81.

Determine the general term (T_n) of the linear number pattern.

$-81 = -5(19) + c$ ✓ **sub T19 = −81**
$c = -81 + 95$
$c = 14$ ✓
$\therefore T_n = -5n + 14$ ✓ (3)

[12]

QUESTION 3

3.1 A bluetooth speaker complete with radio is offered to Mr Broccardo for R3 900.
He cannot afford the deal so agrees to a **hire purchase** agreement at 9,8% per annum and is required to make equal monthly repayments for **two** years.
Determine Mr Broccardo's monthly instalments.

$A = P(1 + in)$
$A = 3\ 900(1 + 9{,}8\% \times 2)$ ✓**sub into correct formula**
$A = 4\ 664{,}40$ ✓
$\therefore \frac{4664{,}40}{24}$ ✓**24**
$= R194{,}35$ per month ✓ (4)

3.2 In the year 2000, a dozen eggs cost R5,00. The price of eggs in 2023 is R48,00 per dozen.
Calculate the annual inflation rate for eggs over the past 23 years.

$A = P(1 + i)^n$
$48 = 5(1 + i)^{23}$ ✓**sub into correct formula**
$i = \sqrt[23]{\frac{48}{5}} - 1$ ✓
$\therefore i = 10{,}33\%$ ✓**A**
(3)

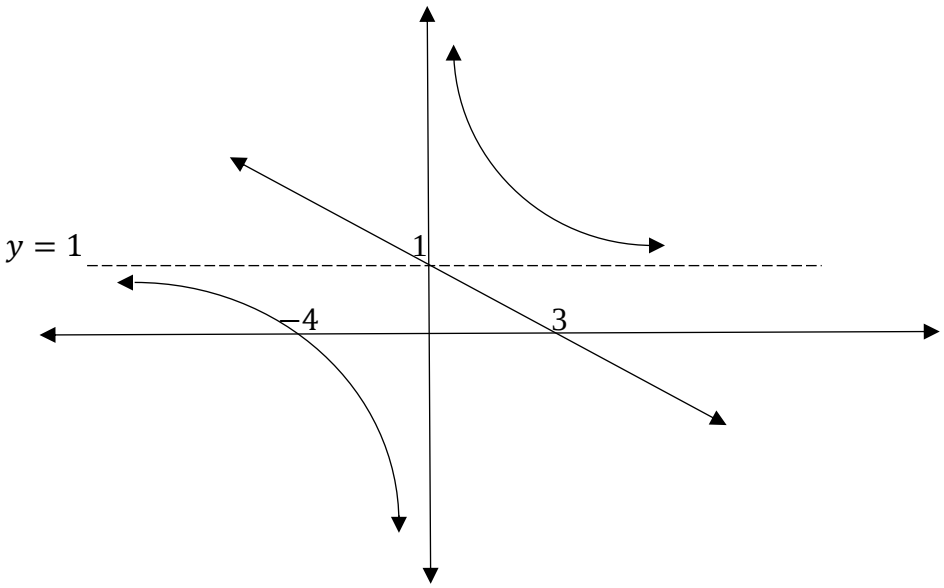
3.3 Lara deposits R5 000 into a unit trust account that offers an interest rate of 10% p.a compounded annually in order to pay for an overseas holiday in 5 years time.
After the second year, the interest rate decreases to 6,5% p.a. compounded annually.
After four years, Lara runs into financial difficulty and withdraws R1 000.
How much will she have available for her holiday?

$A = 5000(1 + 10\%)^2$ ✓**A** $(1 + 6{,}5\%)^3$ ✓**A** $- 1000(1 + 6{,}5\%)^1$ ✓**A**
$= R\ 6\ 243{,}10$ ✓ (4)

[12]

QUESTION 4

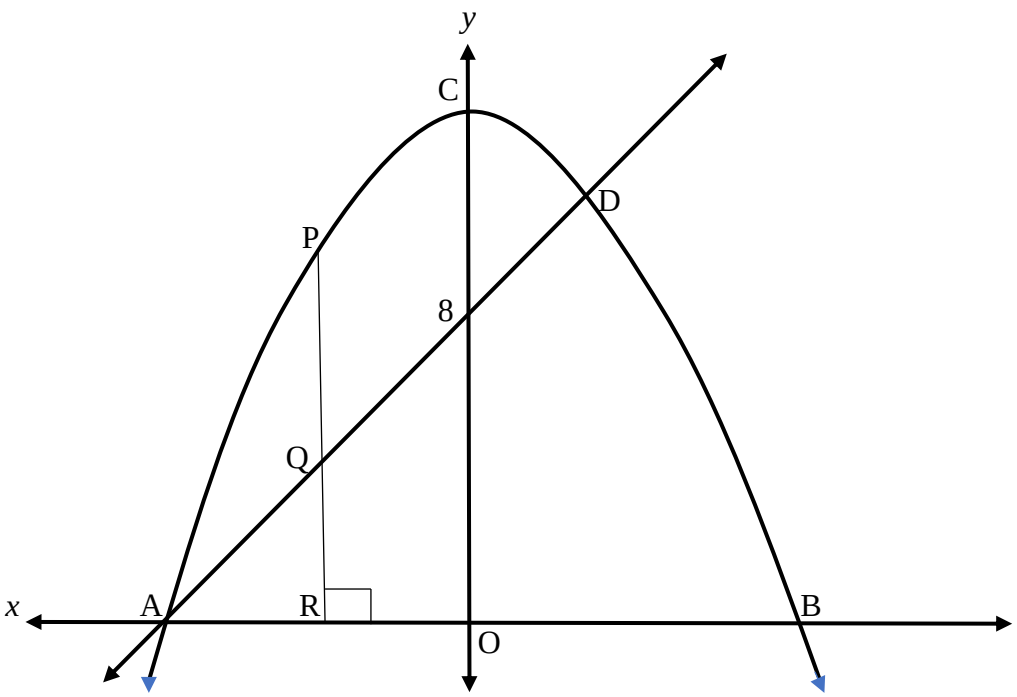
4.1 Sketch the graphs of $p(x) = \frac{4}{x} + 1$ and $h(x) = -\frac{1}{3}x + 1$ on the annexure provided at the end of the question paper. Clearly show ALL asymptotes and intercepts with the axes.



(5)

Straight line: ✓y-int ✓x-int
Hyperbola: ✓asymptote ✓x-int ✓shape in correct quadrants

4.2 The graphs of $f(x) = -x^2 + 16$ and $g(x) = mx + 8$ are sketched in the diagram below. A and B are the $x-$ intercepts of f. C is the turning point of f.



4.2.1 Write down the length of OC.

OC = 16 units ✓ (1)

4.2.2 Determine the length of AB.

$0 = -(x^2 - 16)$
$0 = -(x + 4)(x - 4)$ ✓
$\therefore x = 4 \text{ or } -4$ ✓
AB = 8 units ✓ (3)

4.2.3 Determine the value of m.

$$m = \frac{\Delta y}{\Delta x}$$

$m = \frac{8}{4} = 2$ ✓ (1)

4.2.4 Show that the coordinates of D, a point of intersection of f and g, is D(2 ; 12).

$-x^2 + 16 = 2x + 8$ ✓ **CA from 4.2.3**
$-x^2 - 2x + 8 = 0$
$-(x^2 + 2x - 8) = 0$ ✓
$-(x + 4)(x - 2) = 0$ ✓
$\therefore x = -4 \text{ or } 2$ ✓
$y = 2(2) + 8$ ✓
$\therefore y = 12$ (5)
$\therefore$D(2 ; 12)

4.2.5 If OR = 1 unit, determine the length of PQ.

$y = 2(-1) + 8$ ✓sub -1 into str line **and** parabola
$\therefore y = 6$ ✓**CA** only if -1 is used

$y = -(-1)^2 + 16$
$\therefore y = 15$ ✓**CA** only if -1 is used

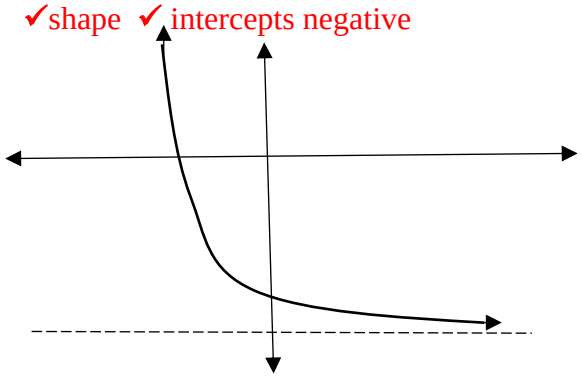
PQ = $15 - 6$
= 9 units ✓**CA** subtraction (4)

4.2.6 For which values of x is $g(x) \leq f(x)$?

$-4 \leq x \leq 2$ ✓✓ **A** (2)

4.3 Draw a rough sketch of $y = a \cdot b^x + q$ if $a > 0$, $0 < b < 1$ and $q < 0$.

✓ asymptote ✓shape ✓intercepts negative



(3)

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QUESTION 5

5.1 Two events, A and B, are such that P(A) $= \frac{1}{2}$ and P(B) $= \frac{1}{3}$.

5.1.1 Calculate P(A and B) if P(A or B) $= \frac{2}{3}$.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$\frac{2}{3} = \frac{1}{2} + \frac{1}{3} - P(A \text{ and } B)$ ✓**A**
$P(A \text{ and } B) = \frac{1}{6}$ ✓**A** (2)

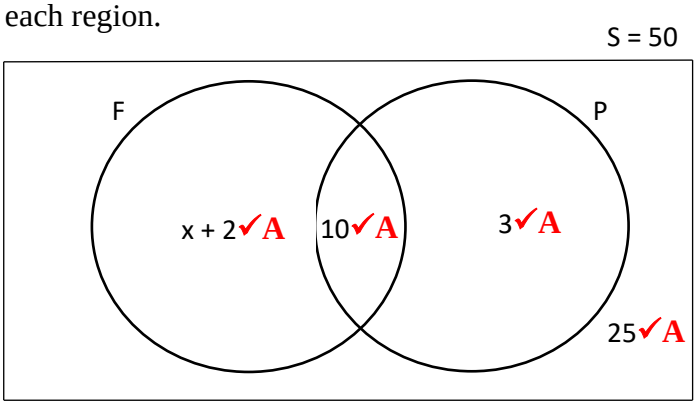
5.1.2 Calculate P(A or B) if A and B are mutually exclusive.

$P(A \text{ or } B) = P(A) + P(B)$
$P(A \text{ or } B) = \frac{1}{2} + \frac{1}{3}$ ✓**A**
$\therefore P(A \text{ or } B) = \frac{5}{6}$ ✓**A** (2)

5.2 In a group of 50 teenagers,

- 10 can play the flute(F) and the piano(P).
- 3 can only play the piano.
- $x + 2$ can play the flute but not the piano.
- 25 can play neither the flute nor the piano.

5.2.1 Represent the information given above in a Venn diagram, showing the number of outcomes in each region.



(4)

5.2.2 Determine the value of *x*.

$x + 2 + 10 + 3 + 25 = 50$ ✓
$x = 10$ ✓

(2)

5.2.3 Calculate the probability that a teenager chosen from this group at random can play:

(a) only the flute.

$P(F\ only) = \frac{12}{50}$ **✓CA from 5.2.2**

(1)

(b) at least one of the two instruments.

$P(at\ least\ one\ instrument) = \frac{12+10+3}{50}$ **✓addition of correct regions**

$= \frac{25}{50}$ **✓ no CA if incorrect regions used to add**

(2)
[12]