WMC 2020 Senior Secondary Qualifying Round Solutions

1. **B** $|-3| - |4| = 3 - 4 = -1$

2. **D** Let the three digit be ABC. Then as C is a square we have $C \in \{0, 1, 4, 9\}$. Which we will deal with case by case.

   If $C = 0$, then $\frac{A}{B} = 0$, which forces $A = 0 = C$ so no solutions of this form.

   Similarly, if $C = 1$, then $\frac{A}{B} = 1$, which forces $A = B$.

   If $C = 4$ then $A = 2B$, this gives candidates 214, 424, 634 and 848. Of which only 214 and 634 are valid.

   Finally if $C = 9$ then $A = 3B$, which gives candidates 319, 629 and 939 of which 319 and 629 are valid.

   Hence there are four solutions.

3. **D** When $x = 0$, y can be any integer between -19 and 19 inclusive, giving us 39 options. When $x = k$ for any $k \in \{1, 2, \ldots, 19\}$ y can be between $-(19 - k)$ and $19 - k$ inclusive, and also since k can be negative we have $39 + 2 \times 37 + 35 + \cdots + 1 = 39 + 2 \times 19^2 = 761$ pairs.

4. **A** Let A and B be the amount of money Alice and Bob have before any giving. This gives us the simultaneous equations.

$$3(B - 3) = A + 3$$

   and

$$2(B + 1) = A - 1$$

   These can be solved by standard methods to get $A = 33$ and $B = 15$

5. **A** This can be seen by doing direct addition.

6. **D** $\sqrt{0.444\ldots} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0.666\ldots$

7. **A** We split this into three cases by the person who missed.

$$\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} = \frac{6}{2 \times 3 \times 4} = \frac{1}{4}$$

8. **E** In the set $S = \{101, 102, \ldots, 299, 300\}$ there are $42 - 14 = 28$ numbers that are divisible by 7, there are $6 - 2 = 4$ numbers that are divisible by 49 and there are no numbers that are divisible by any power of 7 beyond that. So $7^{28+4} = 7^{32}$ is the largest power of 7 that divides $101 \times 102 \times \cdots \times 299 \times 300$.

9. **C** The plane, starts off as a single region. The first line can add an additional region the second line two additional regions (if it crosses the first line). The third line can add an addition three regions (crossing both lines), and so on. So finally $1 + 1 + 2 + 3 + 4 + 5 = 16$.

10. **C** The smallest number whose digits add up to anything must have as few digits as possible. So most (all but the first) of these digits should be 9. In the case of 2020 this means 49999...999 as 2020 is congruent to 4 modulo 9.

11. **A** The only number that works is 105. Let the number have decimal representation ABC. Then $100A + 10B + C = 7(10A + C)$ which reduces to $30A + 10B = 6C$. The left hand side is a multiple of 5 so C is either 0 or 5. If $C = 0$ we have $30A + 10B = 0$, which puts $A = B = 0$ and A cannot be 0 or ABC is not a true three digit number. So we have $C = 5$ and $30A + 10B = 30$ which forces $A = 1$ and $B = 0$.

12. **B** Set $DE = EA = x$, applying the cos law to triangle BDE (the length of BE is $15 - x$ and $\angle DBE$ is 60^o) to compute $x = 7$. Applying the cos law again to triangle BCE gives $CE = 13$.

13. **B** The length of the two shortest rods is at least 1. For minimality we set them both to equal 1. Now the third shortest is $1 + 1$ (by the triangle inequality). Similarly the fourth shortest rod is at least $2 + 1 = 3$. This forms a pattern of the Fibonacci numbers and so the longest rod is at least the $10th$ Fibonacci number which is 55.

14. **A** We write 20! as the product of prime of prime factors. By considering how many numbers less than or equal to to 20 have each prime as a factor we can get $20! = 2^{18} \times 3^8 \times 5^4 \times 7^2 \times 11 \times 13 \times 17 \times 19 = 2^{14} \times 3^8 \times 7^2 \times 11 \times 13 \times 17 \times 19 \times 10^4$. So the rightmost digit is considering each of these powers except the 10 in mod 10 which gives $4 \times 1 \times 9 \times 3 \times 7 \times 9 \equiv 4 \pmod{10}$.

15. **C** Throughout we will use that if a digit is *unmoved* it must be in its original position.
If the first number is 5 then no value will be *unmoved* and we can arrange the remaining four digits in $4! = 24$ ways.
Now if the second number is 5 then the only way we can have a value that is *unmoved* is if the first number is 1, the rest leave all the numbers *unmoved*. So here, there are $4! - 3! = 18$ ways.
Now suppose 5 is the third number. Then if 1 is the first number, it will be *unmoved*. So 1 cannot be the first number. Now if 1 is the second number we cannot have that $1, 2, 3, 5$ are *unmoved* and so 4 is the only value that can be *unmoved* which cannot happen since 5 would be to the left of 4. So here there are $4! - 3! = 18$ ways.
Now suppose the first number is fourth number is 5. If 1 is the first number, again it is *unmoved* so we omit all such numbers and get $4! - 3! = 18$ numbers. Now if 1 is the second number, then $1, 2, 4, 5$ are *unmoved* and the only way 3 can be *unmoved* is when we have 21354, so we must subtract this case. If the third number is 1 then no value can remain *unmoved*.
So in total we have $24 + 18 + 18 + 18 - 1 = 77$ such rearrangements.