



**2020 Wits Mathematics Competition
Qualifying Round
Grades 10, 11 and 12**

Instructions

This exam consists of 15 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The first 5 questions are each worth 3 points, the next 5 questions are each worth 4 points and the last 5 questions are each worth 5. The total number of points available is 60. The time limit on this exam is 75 minutes, calculators and geometric implements may NOT be used. If you are using the computer friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

“The really unusual day would be one where nothing unusual happens.”. — Persi Diaconis

SHARP

A. 3 point questions

1. The absolute value function is defined by $|n| = \begin{cases} n, & \text{if } n \geq 0, \\ -n, & \text{if } n < 0. \end{cases}$

What is the value of $|-3| - |4|$?

- A. -7
 - B. -1
 - C. 1
 - D. 7
 - E. None of the above.
2. Aaliyah has a three-digit code for a padlock. She has forgotten the code but she knows that all three digits are different. She also knows that if you divide the first digit by the second digit and square the result you get the third digit. How many possible codes are there?
- A. 1
 - B. 2
 - C. 3
 - D. 4
 - E. 5
3. Determine the number of pairs of integers $(x; y)$ such that $(x, y): |x| + |y| < 20$. Where $|x|$ is the absolute value function defined in question 1 above.
- A. 400
 - B. 600
 - C. 661
 - D. 761
 - E. 790
4. If Bob gave Alice $R3$, then Alice would have three times as much money as Bob. If instead Alice gave Bob $R1$, then Alice would have twice as much money as Bob. What is the ratio of Alice's money : Bob's money before any giving happens?
- A. 11 : 5
 - B. 9 : 4
 - C. 9 : 5
 - D. 3 : 2
 - E. 5 : 3

5. What is the sum of $111111111 + 22222222 + 3333333 + 444444 + 55555 + 6666 + 777 + 88 + 9$ (i.e nine one's in a row, eight two's in a row and so on until a single nine)?
- A. 137174205
 - B. 37174205
 - C. 134174205
 - D. 337174205
 - E. 137274205

B. 4 point questions

6. The expression $\sqrt{0,444\dots}$ is written as a decimal. Find the 100th digit.
- A. 0
 - B. 2
 - C. 4
 - D. 6
 - E. 8
7. Three hunters can hit a target with respective probabilities of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. If they all shoot the target once, what is the probability that exactly two of them hit the target?
- A. $\frac{1}{4}$
 - B. $\frac{1}{3}$
 - C. $\frac{3}{8}$
 - D. $\frac{5}{12}$
 - E. $\frac{1}{2}$
8. $101 \times 102 \times \dots \times 299 \times 300 = 7^n \times k$. For natural numbers n and k . Determine the maximum possible value of n .
- A. 26
 - B. 42
 - C. 49
 - D. 30
 - E. 32

9. Given an infinite plane, what is the maximum number of pieces you can split it up into with 5 straight lines?
- A. 6
 - B. 12
 - C. 16
 - D. 8
 - E. 20
10. What is the first digit of the smallest positive whole number whose digits add up to 2020?
- A. 2
 - B. 3
 - C. 4
 - D. 5
 - E. 6

C. 5 point questions

11. When the tens digit of a 3-digit number is removed, the remaining 2-digit number becomes 7 times smaller than the original number. What is the sum of digits of this 3-digit number?
- A. 6
 - B. 7
 - C. 10
 - D. 13
 - E. 17
12. ABC is an equilateral triangle with $AB = 15\text{cm}$. D is a point on BC where $BD = 5\text{cm}$. E is a point on AB where $AE = ED$. Find the length of CE .
- A. 12
 - B. 13
 - C. 14
 - D. 15
 - E. 16
13. Thabo is given ten rods. Each rod has a different whole number length. He finds that he cannot construct a triangle with positive area using any three of the rods. What is the smallest possible length for the longest of the ten rods?

- A. 34
B. 55
C. 64
D. 89
E. 128
14. What is the units digit when all the zeros at the end of $20!$ are removed?
Here $n! = 1 \times 2 \times 3 \dots \times n$.
- A. 4
B. 5
C. 6
D. 7
E. 8
15. Consider a rearrangement of the values of $\{1, 2, 3, 4, 5\}$. A value is called "unmoved" if it is both smaller than all values to its right and larger than all values to its left.

For example, the 3 in $\{2, 1, 3, 5, 4\}$ is unmoved as is the 1 in $\{1, 5, 3, 2, 4\}$.

How many rearrangements of $\{1, 2, 3, 4, 5\}$ have no unmoved values?

- A. 44
B. 76
C. 77
D. 96
E. 45