2020 Wits Mathematics Competition
Qualifying Round
Grades 8 and 9

Instructions

This exam consists of 15 multiple choice questions. There is one correct answer to each question. There is no penalty for incorrect answers. The first 5 questions are each worth 3 points, the next 5 questions are each worth 4 points and the last 5 questions are each worth 5. The total number of points available is 60. The time limit on this exam is 75 minutes, calculators and geometric implements may NOT be used. If you are using the computer friendly answer sheet you should fill it in in BLACK pen (other colours do not scan well). Time may be given for filling in name, school and other personal details.

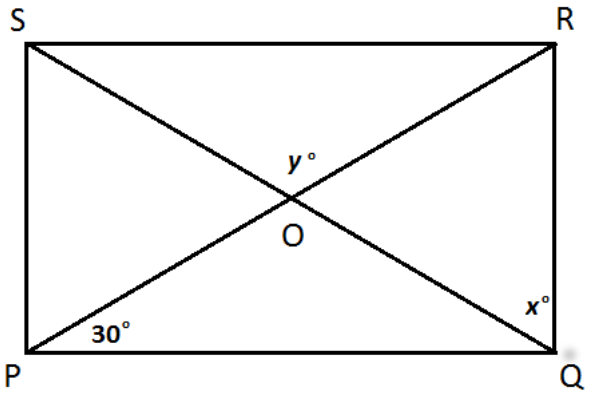
"The really unusual day would be one where nothing unusual happens.". — Persi Diaconis

A. 3 point questions

1. The three digit number $5A6$ is divisible by 3 and 4. The digit A can be:

   A. 2
   B. 3
   C. 4
   D. 7
   E. 8

2. In the given figure, $PQRS$ is a rectangle. Find the value of $x + y$.



   A. 90^o
   B. 120^o
   C. 135^o
   D. 150^o
   E. 180^o

3. It is said that you can only fold a piece of paper in half 7 times. Harold folds a sheet of paper in half 5 times and then makes a hole in the folded paper. How many holes does the sheet of paper have after it is unfolded again?

   A. 32
   B. 36
   C. 25
   D. 50
   E. 64

4. If $x < -1$ which of the following expressions hold?

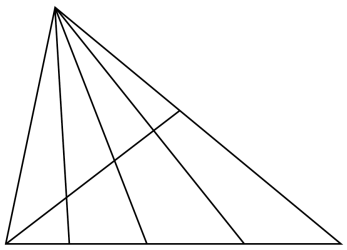
   A. $x < x^2 < x^3$
   B. $x < x^3 < x^2$
   C. $x^2 < x^3 < x$
   D. $x^3 < x^2 < x$
   E. $x^3 < x < x^2$

5. What is the smallest integer value of n such that $2^n > 2020$?
   A. 12
   B. 20
   C. 11
   D. 10
   E. 1010

B. 4 point questions

6. Pythagorean primes are prime numbers that are the sum of two squares. For example, $5 = 2^2 + 1^2$. How many Pythagorean primes are there below 50?
   A. 3
   B. 4
   C. 5
   D. 6
   E. 7

7. The number of distinct (different) triangles in the figure is...



   A. 20
   B. 25
   C. 15
   D. 21
   E. 24

8. A whole number is called an increasing number if each digit in the number is greater than the digit to its left. How many increasing numbers are between 5000 and 10000?
   A. 5
   B. 6
   C. 7
   D. 10
   E. More than 10

9. Rectangle $ABCD$ has an area of $192m^2$ and a perimeter of $56m$. What is the difference between the length and the width of this rectangle?

   A. 2

   B. 4

   C. 6

   D. 8

   E. 10

10. A shop assistant is told to decrease the price on all shoes in the store by 25%. By mistake the clerk increases the price by 25%. What percentage discount should now be given to sell the shoes at the price the manager intended?

   A. 50%

   B. 60%

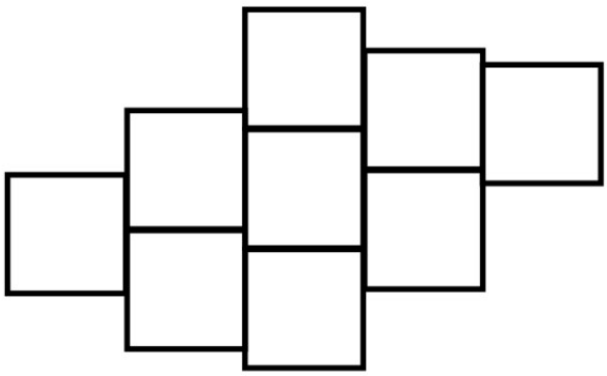
   C. 20%

   D. 40%

   E. 75%

C. 5 point questions

11. If each square has a side length of $1cm$, what is the perimeter of the shape?



   A. Impossible to determine

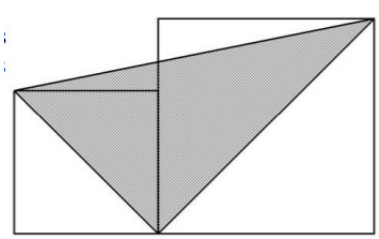
   B. $16cm$

   C. $15cm$

   D. $17cm$

   E. $14cm$

12. Two squares are adjacent to each other as shown. One has sides of length $5cm$ and the other has sides of length $7cm$. The area of the shaded region is:



A. $35cm^2$
B. $35.5cm^2$
C. $36cm^2$
D. $36.5cm^2$
E. $37cm^2$

13. At a family reunion of 12 people, the official photographer takes a picture of two people at a time. If each person has his picture taken with each of the other people, the minimum number of pictures that could be taken is:

A. 66
B. 60
C. 120
D. 132
E. 144

14. Three hunters can hit a target with respective probabilities of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$. If they all shoot at the target once, what is the probability that exactly two of them hit the target?

A. $\frac{1}{4}$
B. $\frac{1}{3}$
C. $\frac{3}{8}$
D. $\frac{5}{12}$
E. $\frac{1}{2}$

15. Thabo is given ten rods. Each rod has a different whole number length. He finds that he cannot construct a triangle with positive area using any three of the rods. What is the smallest possible length for the longest of the ten rods?

A. 34
B. 55
C. 64
D. 89
E. 128