Wits Mathematics Competition
Grade 10-12 **9 May 2018**
Time Limit: 75 Minutes
Full Name: _____
E-mail: _____
Seat Number: _____
School: _____
Grade: _____

Instructions

This exam consists of 3 sections. Section A contains 10 multiple choice questions for 3 marks each. Section B consists of 10 single answer questions for 5 marks each. Section C consists of two questions which require full workings, each for 10 marks. You should answer Sections A and B on this page and section C on the sheets the questions are printed on.

Scores

Section	Mark	Perfect
A		30
B		50
C		20
Total		100

Problems worthy of attack prove their worth by fighting back - Piet Hein.

Section A [30 Marks]

Multiple Choice Questions					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

Section B [50 Marks]

Single Answer Questions	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

A. Multiple Choice

1. Compute the value of $2016 - 2017 + 2018 - 2019 + 2020$

   A. 2016

   B. 4024

   C. 90

   D. 2018

   E. 2021

2. If If $a = \frac{1}{2}$ and $b = \frac{3}{2}$, find the value of $\frac{6a+18b}{12a+6b}$.

   A. 9

   B. 7

   C. 6

   D. 10

   E. 2

3. How many integers are greater than $\frac{2}{7}$ and less than $\frac{31}{4}$?

   A. 3

   B. 4

   C. 7

   D. 8

   E. 12

4. Find the sum of the first 2018 terms of the sequence $1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, ...$

   A. 5048

   B. 5012

   C. 5043

   D. 5029

   E. 5061

5. Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?

   A. P^2Q

   B. P^nQ^m

   C. P^nQ^{2m}

   D. $P^{2m}Q^n$

   E. $P^{2n}Q^m$

6. x and y are distinct positive integers less than 20. If $y = x^3$ determine the value of y^2.
   A. 16
   B. 25
   C. 36
   D. 49
   E. 64

7. John ate 120 peanuts over four consecutive nights. Each night (except the first) he ate six more than the previous night. How many did he eat on the last night?
   A. 30
   B. 33
   C. 36
   D. 39
   E. 42

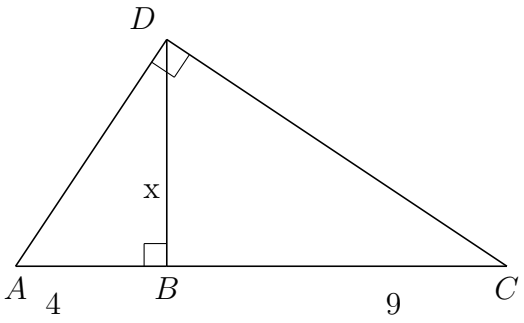
8. Which number must be removed from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ so that the average of the remaining ten is $6, 1$?
   A. 1
   B. 2
   C. 3
   D. 5
   E. 8

9. Seven children are born on the same day of the year in seven consecutive years. The sum of the ages of the youngest three is 42. How old is the oldest?
   A. 16
   B. 17
   C. 18
   D. 19
   E. 20

10. If 40% of P is 10% of Q, what percentage of P is Q?
    A. 18%
    B. 22%
    C. 25%
    D. 26%
    E. 30%

B. Single Answer

11. Find the value of x, if $A\hat{D}C$ and $A\hat{B}D$ are right angles.



12. A rectangle with diagonal length x is twice as long as it is wide. What is the area of the rectangle, in terms of x?

13. Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and granddaughters have no daughters?

14. Find the number of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ that are subsets of neither $\{1, 2, 3, 4, 5\}$ nor $\{4, 5, 6, 7, 8\}$.

15. An octagonal swimming pool has sides which are, consecutively 10 m, 20 m, 30 m, 40 m, 50 m, 60 m, 70 m and 80 m. All the pool's angles are right angles. Find the top surface area of the pool, in square metres.

16. Points B and C lie on $\overline{AD}$. The length of $\overline{AB}$ is 4 times the length of $\overline{BD}$, and the length of $\overline{AC}$ is 9 times the length of $\overline{CD}$. What fraction of the length of $\overline{AD}$ is the length of $\overline{BC}$?

17. An orange is 80% water (by mass). If 75% of the water is evaporated, what percentage of the orange is now water?

18. Find the number of positive integers less than or equal to 2018 whose base-three representation contains no digit equal to 0.

19. How many zeros does the number $55! + 60! + 65!$ end with?

20. Find in terms of $p > 1$ the value of the following fraction:

$$\frac{1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + ..}{1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + ..}$$

C. Proof Questions

21. A 10×10 square grid of light bulbs is initially off. For every 3×3 connected sub-grid you have a switch which will toggle every light bulb within that grid. That is, it will change the bulbs state, either from on to off or off to on. You have a similar switch for every 5×5 sub-grid. Is it possible to get any arrangement of on/off bulbs by flipping these switches? Provide a proof.

22. Which members of the sequence 101,10101,1010101,101010101,... are prime? Provide a proof.