

WMC Junior Secondary 2018 Solutions

Section A

1. **B** $29 + 14 + 21 = 64 = 2^6$
2. **E** $\frac{x-2y}{x+y} = \frac{4-2(-3)}{4-3} = 10$
3. **A** $25\% = 5 \times 5\%$ and so 25% of the number is $5 \times 8 = 40$.
4. **D** Taking reciprocals gives $x + \frac{1}{5} = \frac{3}{5} \implies x = \frac{2}{5}$.
5. **D** $\sqrt{36}\sqrt{16} = \sqrt{36 \times 4} = 6 \times 2 = 12$.
6. **C** $x = 36$ and so $\frac{x}{4} = \frac{36}{4} = 9$
7. **B** The water weighs $2 \times (800 - 500) = 600$ g and so the glass weighs $800 - 600 = 200$ g.
8. **E** The length in metres of each step is $\frac{500}{625} = \frac{4}{5}$ and so in 10000 steps, the distance she covers in metres is $10000 \times \frac{4}{5} = 8000$ m = 8 km.
9. **E** $8362 \div 12 = 696$ remainder 10.
10. **C** First we must choose two of the three coins to land on heads, then each of those have a $\frac{1}{2}$ chance of landing on heads or tails. So the probability is $\binom{3}{2} \times \left(\frac{1}{2}\right)^3 = \frac{3}{8}$

Section B

11. **15** $\frac{500}{35} = 14\frac{10}{35}$ and so the minimum number of bottles she should buy is 15.
12. **6** Pythagoras on $\triangle ABD$ and $\triangle CBD$ gives $AD = \sqrt{x^2 + 16}$ and $CD = \sqrt{x^2 + 81}$. Then Pythagoras on $\triangle ACD$ gives $x^2 + 16 + x^2 + 81 = 13^2 \implies x = 6$.
13. **216** Let s be the side length (in cm) of the cube. Then $6s^2 = 96 \implies s = 6$. So the volume of the cube is $s^3 = 6^3 = 216\text{cm}^3$.
14. **25** Let x be the number of marbles that Tyrone gave to Eric. Then Tyrone now has $97 - x$ marbles and Eric has $11 + x$ marbles. Then $97 - x = 2(11 + x)$ and solving gives $x = 25$.
15. **5** This can be achieved when there are two daughters and three sons. Now there cannot be any less girls since if there is only one girl, she has no sister. Similarly, there cannot be any less boys since each boy has at least two brothers.

16. **144** Let r be the radius of the circle. Then $2\pi r = 24 \implies r = 12$. Then the area of the circle is $\pi r^2 = \pi \times 12^2 = 144\pi$.
17. **24** Clearly a must be divisible by 3. Also since b is positive, it must be that $a \leq 5$ and so a can only be 3. This gives that $b = 8$ and so $ab = 24$.
18. **36** Let p be the full price of a movie ticket. Then $\frac{3}{4}x = 6 \implies x = R8$. So the youngest generations pays $R4$ each, the middle $R8$ each and the oldest $R6$ each so in total they pay $2(4 + 8 + 6) = R36$.
19. **196** Consider the sets $\{1, 2, 3\}$ and $\{6, 7, 8\}$. We must choose at least element from both of these sets, which we can do in $2^3 - 1 = 7$ ways. Now for the set $\{4, 5\}$ we can choose any (including the empty set) subset so we have $2^2 = 4$ choices. So the total number of such subsets is $7 \times 7 \times 4 = 196$.
20. **222** 2018 in base 3 is 2202202. So consider $k = 1, 2, 3, 4, 5, 6$. Then there are 2^k trinary numbers of length k that satisfy the condition since each of the digits can be 1 or 2. Then suppose the trinary number has seven digits. Then if the first digit is 1 the rest of the numbers can be any of one and two so we get $2^6 = 64$ numbers, and if the first digit is 2, the next has to be 1 or it is too large, and so the rest of the 5 can be filled in $2^5 = 32$ ways. So in total we have $2 + 4 + 8 + 16 + 32 + 64 + 64 + 32 =$ numbers.

Section C

Question 21

We can first take two sets of three coins each and put them on the scale. If they weigh the same, we know that the fake coin is one of the remaining coins, and on the second use of the scale, we can put the two coins on either side and see which one is lighter and is the fake coin. Now if the two sets of three coins do not weigh the same, we know that the fake coin must be in the lighter set. Then in the second use of the scale, we can put any two of those three coins. If they weigh the same, the other coin must be the fake coin, otherwise the lighter coin is the fake coin.

Question 22

The polygon can be uniquely formed as shown in the diagram. The area is then equal to $80 \times 100 - (10 \times 80 + 50 \times 40) = 5200$

